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APRIL 1964 AIAA JOURNAL VOL 2, NO 4

Lift on an Oscillating Ellipsoid of Revolution

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A theoretical and experimental investigation of the lift and moment on an $\frac{8}{1}$ ellipsoid of revolution performing small heaving oscillations has been carried out The maximum circulatory lift coefficient measured was 0 035, about half the value obtained in steady flow at the same maximum angle of attack and Reynolds number Also, a phase lag of 20° in the buildup of the lift was observed The maximum pitching moment was reduced by 16% from the value predicted by potential theory The circulatory lift was calculated theoretically by estimating the net vorticity discharged into the main body of the fluid from the boundary layer at any instant of time This component of lift is caused by a vector circulation on a control surface surrounding the body and its wake The total lift was obtained by adding to the circulatory lift two additional components that depend on acceleration and are associated with the "apparent mass" of the body

Nomenclature

A	= amplitude of oscillation of the model
a	= half-length of major axis of the model
c_p	= pressure coefficient = $(p - p)/(\frac{1}{2}\rho V_\infty^2)$
h_i	= metric coefficients (i = 1, 2, 3)
i,**j**,**k**	= unit vectors
n	= unit normal
p	= pressure
RN	= Reynolds number = $V_\infty a/\nu$
t	= time
U,W,V	= velocity components of the external flow in directions x_i (i = 1, 2, 3, respectively)
$\mathbf{V}_\infty$	= freestream velocity
(s,z,y)	= coordinate system along streamlines and equipotential lines of the mean external flow, y being normal to the surface of the body
(x_1,x_2,x_3)	= orthogonal curvilinear coordinates; x_1 and x_2 are body surface coordinates, x_3 is normal to the surface
δ	= boundary-layer thickness
ν	= kinematic viscosity of fluid
ρ	= density of fluid
ω	= frequency of oscillation of the model
Ω	= angle of meridian from vertical plane

Subscripts

1	= fluctuating component
u,w,v	= velocity components in boundary layer in directions x_i (i = 1, 2, 3, respectively)
δ^*	= displacement thickness of the boundary layer
θ	= momentum thickness of the boundary layer

Introduction

IN Ref 1 a general theory concerning the forces acting on a body in incompressible, viscous, unsteady flow was presented It was shown that the instantaneous lift force could be predicted in terms of a vector circulation on an arbitrarily large control surface surrounding the body and its wake and that, in addition, two terms proportional to acceleration had to be included

The present paper describes an experimental investigation on a slender ellipsoid of revolution performing small heaving oscillations in a steady stream The objectives of these tests consisted in measuring instantaneous lift forces and pitching moments caused by viscous effects and establishing the regions on the body where actual pressures differ in magnitude and phase from those predicted by inviscid flow theories

In order to relate the experimentally measured values of the lift with the theory outlined in Ref 1, it is necessary to estimate the net flux of vorticity from the boundary layer into the main stream at each instant of time This problem was generally discussed in Ref 1, and expressions for the transport of vorticity in both steady and unsteady flow were given These equations were justified by experimental observations made in steady flow on slender ellipsoids of revolution which indicated a sharp increase in vorticity in the main stream in the vicinity of the separation line Our present task, therefore, consists in predicting the instantaneous locations of the separation line on our test specimen The difficulties of this problem need hardly be emphasized The tests were conducted at a Reynolds number of $2\,7 \times 10^6$, which implies a turbulent boundary layer Among the many unknowns of the problem, for instance, are the effects of the heaving oscillations of the model on transition and separation Nevertheless, the results of an investigation made by Karlsson[2] on a turbulent boundary layer subjected to a periodic oscillation of the external flow indicate that predictions based on a laminar boundary layer will provide a good estimate of the phase lags involved, although not of the magnitude of the lift Karlsson's work showed that, at the frequency at which our model was oscillated, the vorticity in the boundary layer is unable to redistribute itself instantaneously, so that the flow in the boundary layer is composed of a steady part corresponding to the mean external flow and a time-varying

Received October 25, 1963; revision received January 17, 1964

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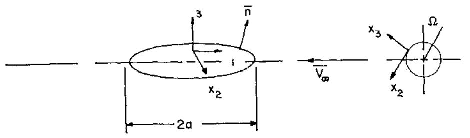


Fig 1 Coordinate system

part very near the wall which is unaffected by the mean external flow but only depends on the oscillations This, however, is also the manner in which a laminar boundary layer is affected Consequently, theoretical predictions for the phase lags may be expected to agree well with test results On the other hand, since the magnitude of the lift depends on the locations of the separation line, a good correlation cannot be anticipated in this case Nevertheless, a prediction based on a laminar boundary-layer analysis appears to be the only possible one at the present time

Laminar Boundary Layer on an Oscillating Ellipsoid of Revolution

The fundamental equations for the three-dimensional laminar boundary layer using an orthogonal curvilinear coordinate system were first established by Hayes[3] He applied Prandtl's simplifications to the full Navier-Stokes equations, expressed in terms of curvilinear coordinates and, in the case of an incompressible fluid, obtained the following set of equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{h_1\partial x_1} + v\frac{\partial u}{h_3\partial x_3} + w\frac{\partial u}{h_2\partial x_2} + \frac{uw}{h_1}\frac{\partial h_1}{h_2\partial x_2} - \frac{w^2}{h_2}\frac{\partial h_2}{h_1\partial x_1} = -\frac{1}{\rho}\frac{\partial p}{h_1\partial x_1} + \nu\frac{\partial}{h_3\partial x_3}\left(\frac{\partial u}{h_3\partial x_3}\right) \quad (1)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{h_1\partial x_1} + v\frac{\partial w}{h_3\partial x_3} + w\frac{\partial w}{h_2\partial x_2} + \frac{uw}{h_2}\frac{\partial h_2}{h_1\partial x_1} - \frac{u^2}{h_1}\frac{\partial h_1}{h_2\partial x_2} = -\frac{1}{\rho}\frac{\partial p}{h_2\partial x_2} + \nu\frac{\partial}{h_3\partial x_3}\left(\frac{\partial w}{h_3\partial x_3}\right) \quad (2)$$

$$\partial p/\partial x_3 = 0 \quad (3)$$

$$\frac{\partial}{\partial x_1}(h_2h_3u) + \frac{\partial}{\partial x_3}(h_1h_2v) + \frac{\partial}{\partial x_2}(h_1h_3w) = 0 \quad (4)$$

where h are the metric coefficients for the curvilinear coordinates We shall now restrict our considerations to a slender body of revolution whose mean angle of attack is 0° We shall choose a coordinate system fixed to the body such that x_1 lies along a meridian (Fig 1) In general, such a change of frame of reference affects the differential equations However, Lighthill[4] has shown that, if the fluid is incompressible, frames of reference are equivalent if they are in a uniform, even though not constant, translational motion relative to each other

The boundary-layer equations may be simplified by means of the following considerations and assumptions:

1) On a body of revolution $\partial h_1/\partial x_2 = 0$ Also the metric coefficient $h_3 \cong 1\ 0$

2) The metric coefficients h_1 and h_2 are independent of x_3, since the radius of curvature of the body surface is large everywhere except possibly near the tips

3) We restrict our analysis to a body performing heaving oscillations in a steady stream such that the instantaneous maximum angle of attack is small, as in the experiments described later The cross-flow component w is zero at the wall and equal to the instantaneous value of the external cross-flow component $W \ll U$ at the edge of the boundary layer The average value of w in the course of a cycle is clearly zero In Eqs (1) and (2) it appears, therefore, possible to neglect the fourth term containing w

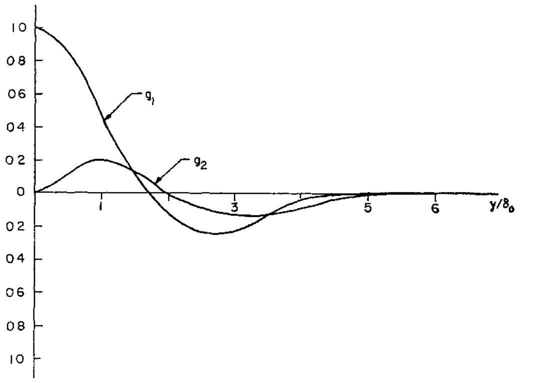


Fig 2 The functions g and g_2 of unsteady laminar boundary-layer theory

As a result of these simplifications, the laminar boundary-layer equations reduce to

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{h_1\partial x_1} + v\frac{\partial u}{\partial x_3} = -\frac{1}{\rho}\frac{\partial p}{h_1\partial x_1} + \nu\frac{\partial^2 u}{\partial x_3^2} \quad (5)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{h_1\partial x_1} + v\frac{\partial w}{\partial x_3} = -\frac{1}{\rho}\frac{\partial p}{h_2\partial x_2} + \nu\frac{\partial^2 w}{\partial x_1^2} \quad (6)$$

$$\partial p/\partial x_3 = 0 \quad (7)$$

$$\frac{\partial}{h_1\partial x_1}(h_2u) + h_2\frac{\partial v}{\partial x_3} + \frac{\partial w}{\partial x_2} = 0 \quad (8)$$

with boundary conditions

$$\begin{aligned} x_3 &= 0: \quad u = v = w = 0 \\ x_3 &= \infty: \quad u = U;\ w = W \end{aligned} \quad (9)$$

We adopt a procedure analogous to that of Lin[5] and separate velocity components and pressure into mean and periodic parts Upon substitution in the boundary-layer equations, we find expressions for the mean flow and the oscillating components

The equation for the average velocity $\langle u \rangle$ is given by

$$\langle u \rangle \frac{\partial\langle u \rangle}{\partial s} + \langle v \rangle \frac{\partial\langle u \rangle}{\partial y} = \nu\frac{\partial^2\langle u \rangle}{\partial y^2} + \langle U \rangle \frac{d\langle U \rangle}{ds} + F \quad (10)$$

where F is a virtual pressure gradient analogous to the Reynolds stress in turbulent flow Note that, in Eq (10), the notation has been simplified by adopting a system of coordinates (s,z,y), where s is measured in the direction of the streamlines of the mean, axisymmetric external flow, z in the direction of the equipotential lines of the mean external flow, and y normal to the surface of the body

As mentioned previously, if the oscillations of the external flow occur at a high frequency, the vorticity in the boundary layer is unable to redistribute itself instantaneously The

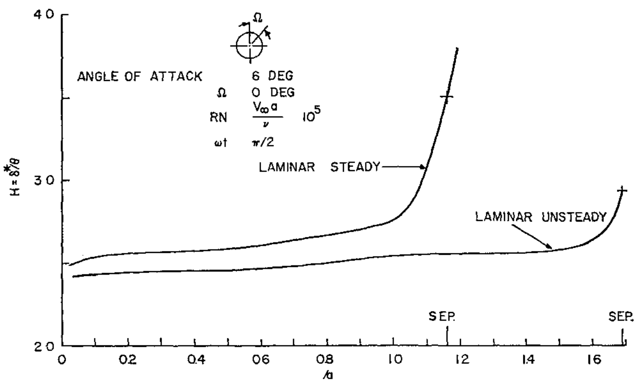


Fig 3 Laminar boundary-layer form factors

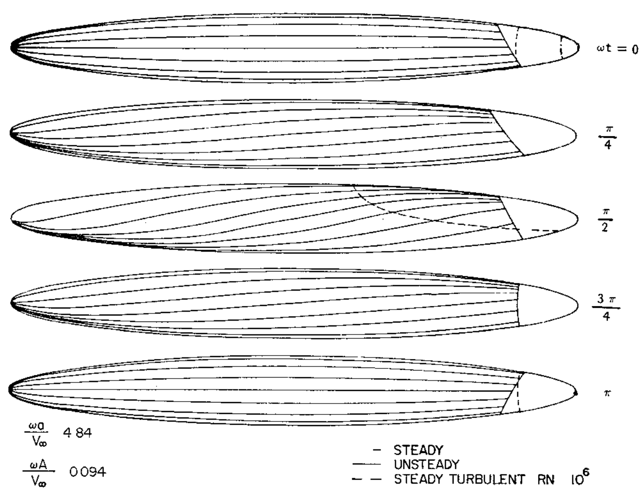


Fig 4 Calculated surface streamlines and lines of separation on an oscillating 8:1 ellipsoid of ıevolution The maximum vertical velocity corresponds to an instantaneous angle of attack of 6° at $\omega t = \pi/2$ The calculated separation lines for steady flow at this angle and in symmetrical flow are also shown

flow in the boundary layer is then composed of a steady part corresponding to the mean flow and a time-varying part very near the wall which is unaffected by the mean flow but only depends on the oscillations Lighthill[4] defined a critical frequency ω_0 such that, when $\omega > \omega_0$, the oscillating motion in the boundary layer is closely approximated by "shear waves" In the case of a flat plate, this critical frequency is given by $\omega_0 \cong 0\ 6\ V_\infty/s$, where s is the distance from the leading edge Clearly, the "high-frequency case" is one of considerable engineering importance In such a case, the three equations available for finding the unknown fluctuating components of the flow in the boundary layer u_1, v_1, and w_1 may be linearized

By estimating orders of magnitude, it may be shown that the only teıms that need be retained are

$$\frac{\partial u_1}{\partial t} = \nu \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial U_1}{\partial t} \tag{11}$$

$$\frac{\partial w_1}{\partial t} = \nu \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial W_1}{\partial t} \tag{12}$$

In the case of harmonic fluctuations of the external flow, suitable solutions of Eqs (11) and (12) satisfying the boundary conditions are of the form

$$u_1 = U_0(1 + e^{-(1+i)y/\delta_0})e^{i\omega t} \tag{13}$$

$$w_1 = w_0(1 - e^{-(1+i)y/\delta_0})e^{i\omega t} \tag{14}$$

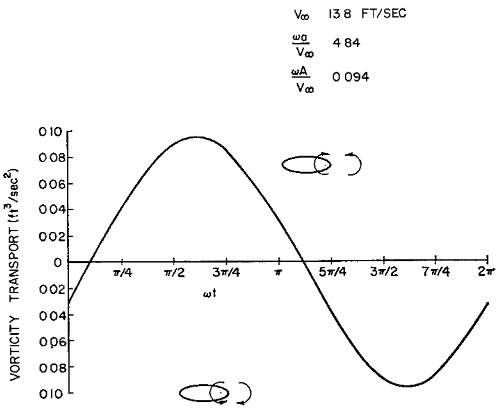


Fig 5 Theoretical net vorticity transport from the laminar boundary layer to the wake

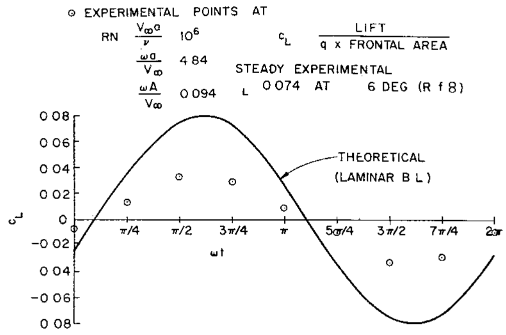


Fig 6 Lift coefficient due to viscous effects

$$v_1 = -\delta_0 \left\{ \frac{y}{\delta_0} + \left(\frac{1}{1+i}\right) e^{-(1+i)y/\delta_0} - \left(\frac{1}{1+i}\right) \right\} \times \left(\frac{U_0}{h_2}\frac{\partial h_2}{\partial s} + \frac{\partial W_0}{\partial z} + \frac{\partial U_0}{\partial s} \right) e^{i\omega t} \tag{15}$$

where $\delta_0 = (2\nu/\omega)^{1/2}$ denotes the "depth of penetration" of the shear waves

In terms of these expıessions, the virtual pressure gradient F is given by

$$F = \frac{1}{2}\, U_0 \frac{\partial U_0}{\partial s}\, g_1 \left(\frac{y}{\delta_0}\right) + \frac{1}{2}\, U_0 \left(U_0 \frac{\partial h_2}{h_2 \partial s} + \frac{\partial W_0}{\partial z} \right) g_2 \left(\frac{y}{\delta_0}\right) \tag{16}$$

where

$$g_1 = e^{-y/\delta_0} \{(2 + y/\delta_0) \cos y/\delta_0 - (1 - y/\delta_0) \times \sin y/\delta_0 - e^{-y/\delta_0}\} \tag{17}$$

$$g_2 = e^{-y/\delta_0} \{y/\delta_0 \cos y/\delta_0 - (1 - y/\delta_0) \sin y/\delta_0\} \tag{18}$$

The function g_1 represents the well-known relationship derived by Lin, but the occurrence of an additional function g_2 is noted (Fig 2) The coefficients of g_2 are due to the three-dimensional nature of the problem and would be zero in a two-dimensional case

Theoretical Results

Some results of this analysis, obtained with the assistance of a digital computer, are shown in Figs 3–6 It is inter-

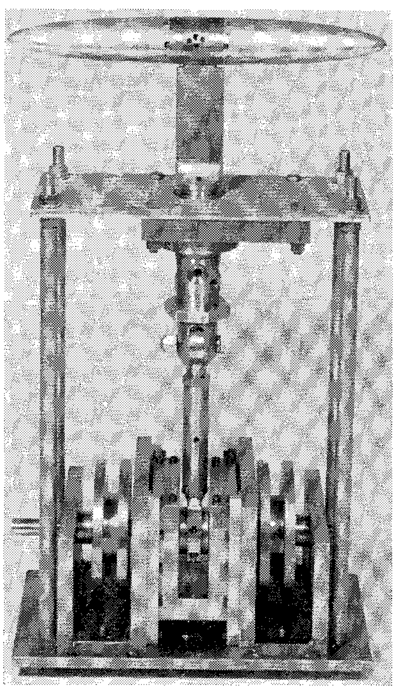


Fig 7 Model and oscillating mechanism

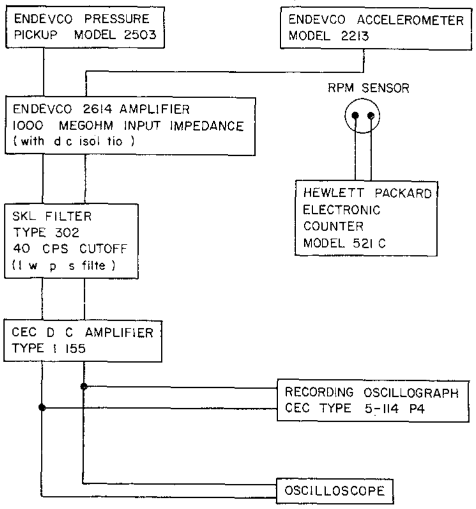


Fig 8 Electrical circuitry

esting to note the differences in boundary-layer characteristics illustrated in Figs 3 and 4 In the former, for instance, the form factor H of the boundary layer in steady and unsteady flow on the top, leeward side of the body is given Note that, in the steady case, the ellipsoid is set at an angle of attack of 6°, whereas in the unsteady case the same angle of attack is reached instantaneously at midstroke In Fig 4, the calculated surface streamlines and the separation lines are shown at various instants of time during a half cycle As is well known, there are important conceptual differences between two- and three-dimensional separation These differences have been discussed by several investigators, notably Eichelbrenner and Oudart [6] The separation lines shown in Fig 4 have been established in accordance with their method and consist of a wall streamline passing through a point of divergence on the top meridian For comparison, the separation lines in steady flow have also been computed, using the method of Ref 7 The results illustrated in Fig 4 not only indicate differences between the steady and unsteady case at the same instantaneous angles of attack but also show that the unsteady separation line anticipates the motion by an angle of $\omega t = \pi/4$

Once the locations of the separation line are known, the flux of vorticity from the boundary layer into the wake can be estimated as discussed in Ref 1 The results of such a calculation are shown in Fig 5 and apply to conditions in the tests The corresponding lift has been found according to Eq (37) of Ref 1 and is compared with experimental results in Fig 6

Experimental Investigation

The objectives of the experiments were to measure instantaneous lift forces and associated pitching moments caused by viscous effects and, in particular, to establish the regions on the ellipsoid where actual pressures differ in magnitude and phase from those predicted by inviscid flow theories

The tests were performed in a water tunnel at a speed of 13 8 fps The model was set at an angle of 0° with respect to the tunnel axis and was oscillated vertically at a frequency of 16 cps and an amplitude of 0 15 in The resulting angle of attack at midstroke was therefore 5 2° The test specimen and the oscillating mechanism are shown in Fig 7 The ellipsoid had a fineness ratio of $\frac{8}{1}$, an over-all length of 16 in, and was geometrically similar to that used by Rodgers[8] in his steady-flow investigations The water-tunnel test section was 12 in in diameter and 30 in long, so that the ratio of model to tunnel diameter also corresponded to those of Ref 8

Hydrodynamic interference between the model and its supporting, streamlined strut was minimized by molding a fillet at their junction Visual observations of the flow made in a wind tunnel by means of smoke showed no detectable vortex formation in this region

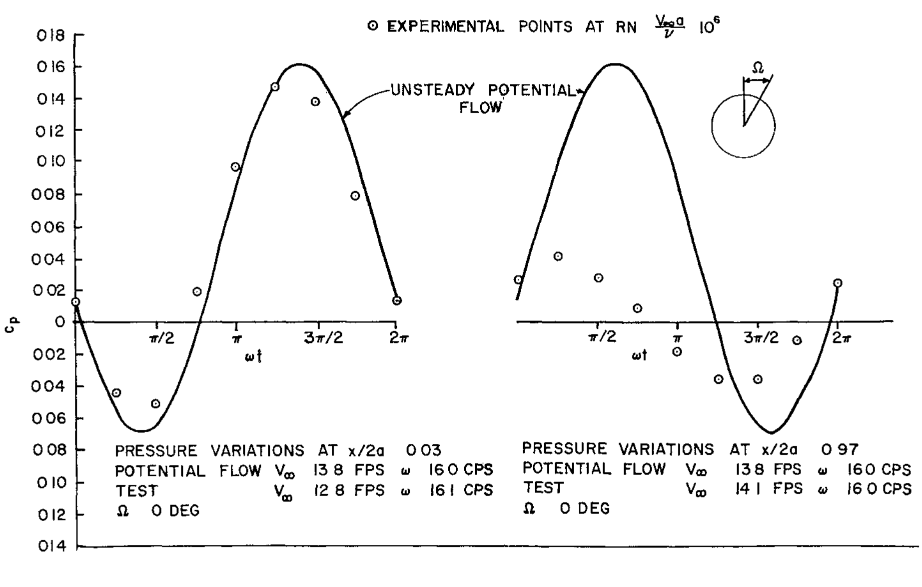
The oscillating mechanism consisted of a shaft with integrally machined eccentric cams that moved the model and suitable counterweights by an equal distance, in opposite phase with each other A small magnet rotating with the shaft generated an electric pulse during each revolution, thus providing an accurate frequency count

Time-dependent pressures were measured by means of five "Endevco" model 2503 piezoelectric transducers placed successively at various points along the body These transducers will accurately resolve dynamic pressures from 0 01 to 10 psi with no response to concurrent static loads as high as 100 psi They have a sensitivity of 130 mv/psi and a frequency response from 2 to 10,000 cps with a 1000-meg load The maximum response of a transducer to vibrations is 0 2 mv/g Figure 8 indicates the electrical circuitry used in the tests

Test Results

Over most of the body, measured pressures agree well with those predicted by potential flow On the rear 20%, how-

Fig 9 Typical pressure variations



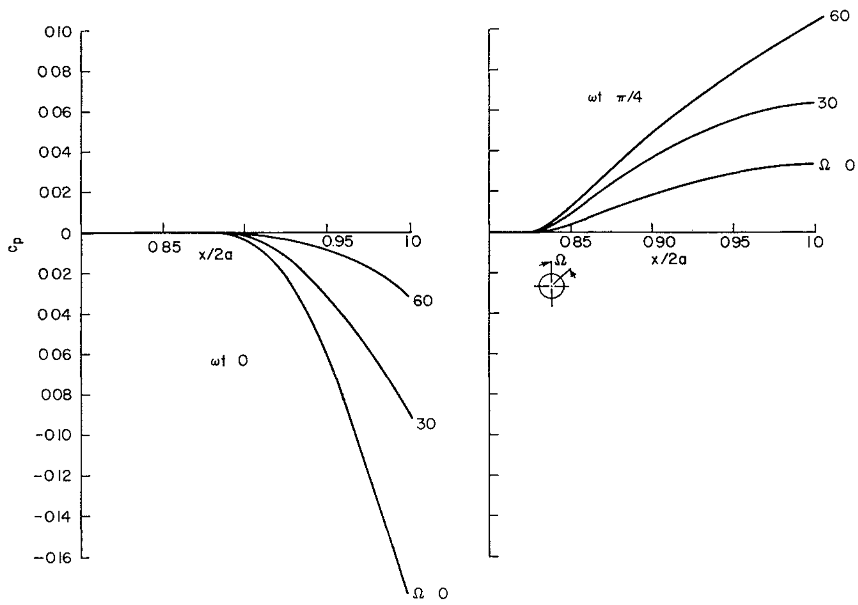


Fig 10 Differences between measured pressures and pressures corresponding to unsteady potential flow

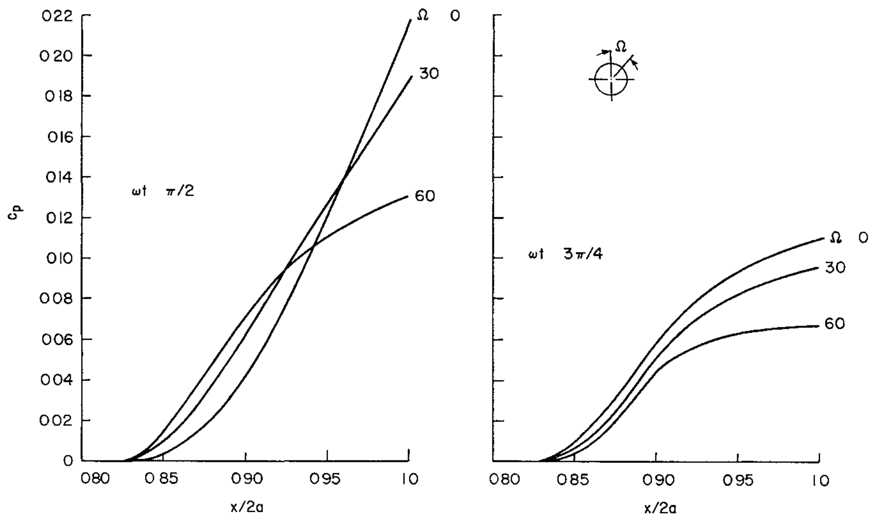


Fig 11 Differences between measured pressures and pressures corresponding to unsteady potential flow

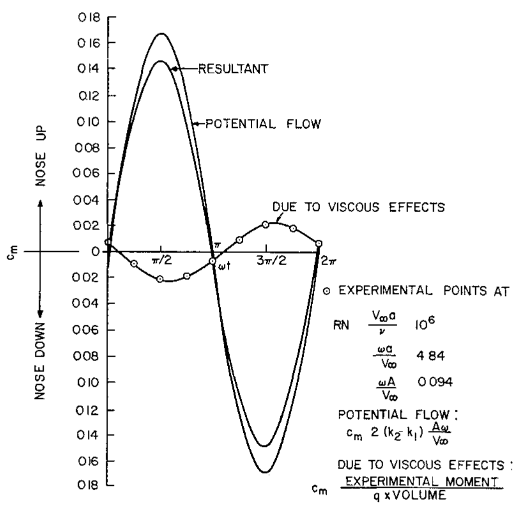


Fig 12 Pitching moment coefficient

ever, actual pressures and those predicted by potential theory or measured in steady flow show a remarkable disagreement (Fig 9) At the top of the stroke ($\omega t = 0$), the vertical velocity of the model is zero In a real fluid under steady-flow conditions, the circulatory lift and pitching moment are zero, since the angle of attack vanishes at that instant It is clear from Fig 10, however, that in unsteady flow there is a downward circulatory lift on the body and a corresponding nose-up pitching moment at this instant of time

At $\omega t = \pi/4$ (Fig 10), the direction of the lift has reversed, and the corresponding pitching moment is nose-down A larger lift is observed at midstroke, $\omega t = \pi/2$, followed by a smaller value at $\omega t = 3\pi/4$ (Fig 11) At the bottom of the stroke ($\omega t = \pi$), there still exists an upward lift and nose-down pitching moment, in contrast with steady-flow observations, which indicate none

Lift and moment coefficients were determined by integration of the differences between observed and inviscid fluid pressures at any instant of time The results are plotted in Figs 6 and 12 The maximum value of the unsteady lift coefficient is 0 035, which compares with 0 074 measured in steady flow on a geometrically similar ellipsoid of revolution and at the same Reynolds number [8] A phase lag of 20° in the buildup of the lift is also noted

The maximum pitching moment is reduced by 16% from the value predicted by potential theory The nose-up pitching moment coefficient at the top of the stroke is of the order of 0 01 At this instant, potential theory predicts no pitching moment at all

Detailed descriptions of the test apparatus, procedure, and experimental results are given in Ref 9

Summary and Conclusions

Theoretical predictions of the unsteady lift based on a laminar boundary layer agree well with the phase but not with the magnitude of experimentally measured values at high Reynolds numbers At the frequencies considered, the fluctuations of the external flow affect a layer very close to the wall of the body, and their influence on a turbulent boundary layer is much the same as on a laminar one This explains the agreement between predicted and measured phase angles The magnitude of the circulatory lift, however, depends on the location of the separation line, and this differs according to the nature of the boundary layer

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